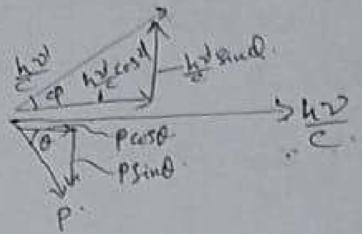
Now. Elastic Collision. 2nd condition. (2)

LAW OF CONSERVATION OF MOMENTUM.

$\Rightarrow$ Momentum of photon + e^- before collision
= Momentum of photon + e^- after collision

ie



NOTE: Photon e^-
$E = h\nu = mc^2$
Also $E = m \times c \times c$
$\therefore E = p \times c$
momentum $p = E/c$
$p = \frac{h\nu}{c}$ (photon)
For e^- $p = m \times vel.$

(2)

$$\underset{\text{(Photon)}}{\frac{h\nu}{c}} + \underset{(e^- \text{ at rest})}{0} = \underset{\text{(Photon)}}{\frac{h\nu'}{c}\cos\varphi} + \underset{(e^-)}{p\cos\theta} \quad \text{(x- direction)}$$

ie $pc\cos\theta = h\nu - h\nu'\cos\varphi$ — (4)

$$\underset{\text{(Photon)}}{0} + \underset{(e^-)}{0} = \underset{\text{(Photon)}}{\frac{h\nu'\sin\varphi}{c}} + \underset{(e^-)}{p\sin\theta} \quad \text{(Y- direction)}.$$

$\Rightarrow$ $pc\sin\theta = h\nu'\sin\varphi$. — (5)

Squaring (4) & (5) and add.

$$p^2c^2 = (h\nu - h\nu'\cos\varphi)^2 + (h\nu'\sin\varphi)^2$$

$$p^2c^2 = h^2\nu^2 - 2h^2\nu\nu'\cos\varphi + h^2\nu'^2\cos^2\varphi + h^2\nu'^2\sin^2\varphi$$

$$p^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\varphi) \quad \text{— (6)}$$

From (1) $mc^2 = h(\nu - \nu') + m_0c^2$

Squaring $\rightarrow$ $m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$ — (7)

Subtracting (6) from (7).

$$m^2c^4 - p^2c^2 = -2h^2\nu\nu'(1 - \cos\varphi) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$\downarrow$$

$$m^2c^4 - m^2v^2c^2 = \quad ''$$

$$\Rightarrow m^2c^2(c^2 - v^2) =$$

note - e^- momentum
$p = m \times v$
where $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

DUAL NATURE OF ~~PARTICLES~~ & RADIATION

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CLASSICAL PHYSICS says RADIATION has WAVE NATURE

Photo Elec. Effect | compton effect → ~~PAR~~ RADIATION Has PARTICLE NATURE

→ RADIATION HAS DUAL NATURE

Expression for Energy of Radiation PLANKS-EINSTIEN Formula for photons (PARTICLE - smallest).

$\left[\lambda = c/\nu \Rightarrow \nu = c/\lambda \right]$ $p = m \times c$

$E = h\nu = mc^2 = mc \times c = p \times c$

$h \cdot \cancel{c}/\lambda = p \times \cancel{c} \Rightarrow \boxed{\lambda = h/p}$

This is for low energy particles with momentum $p = \sqrt{2mE_K}$

For high Energy particles : $\because$ v of particle approaches c its mass $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ increases as v approaches c

Therefore $\boxed{\lambda = hc/E_K}$. [NOTE - with increase in mass & v λ decreases & cannot be measured]

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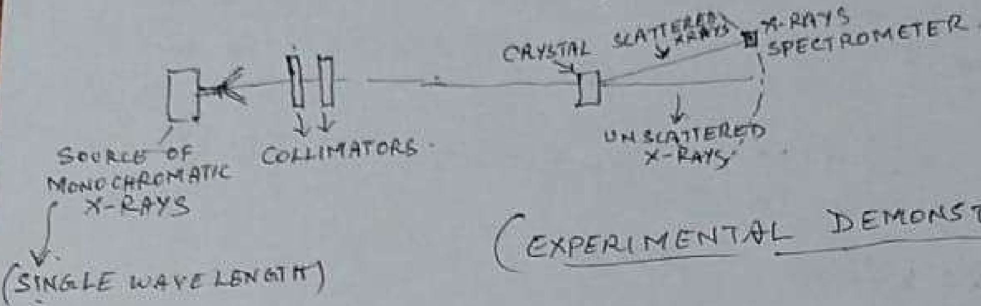
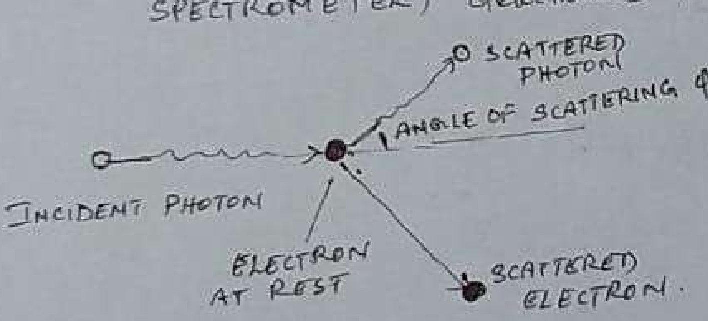
Compton Effect

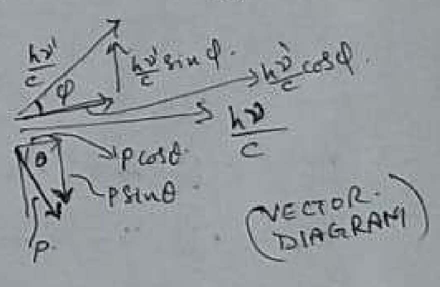
In this effect x-rays (photons) incident on an electron (in a crystal) collide – undergo elastic collision and the photon (x-ray radiation) gets scattered. The observed change in ~~freq~~ (ν) | wavelength (λ) of the incident radiation (x rays | photon) is called Compton effect



(EXPERIMENTAL DEMONSTRATION)

EXP → A Beam of monochromatic x-rays of known wavelength are directed (made incident on) on a target (CRYSTAL | ELECTRON). The scattered x rays at various angles ϕ (observed on X-RAY SPECTROMETER) determines their changed wavelength.





(VECTOR DIAGRAM)

Elastic collision between photon (X-RAY) & electron (CRYSTAL)

LAW OF CONSERVATION OF ENERGY ⇒ Loss of energy of photon = Gain of Energy of e^-

Total Energy of system before collision = Total Energy of sys. after collision

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = h(\nu - \nu') + m_0c^2 \quad \text{—— (1)}$$

$$(h\nu - h\nu') = (mc^2 - m_0c^2)$$

NOTE

Here $h\nu$ = energy of photon before collision.

m_0c^2 = " " e^- "

$h\nu'$ = Energy of photon after collision. & mc^2 = energy of e^- after collision.

(3)

$$\frac{m_0^2c^2}{[\sqrt{(1-v^2/c^2)}]^2}(c^2-v^2) = \text{---}$$

$\Rightarrow \cancel{m_0^2c^4} = \cancel{2h^2 v v' c}$

$$\frac{m_0^2c^2}{\left[\sqrt{\left(\frac{c^2-v^2}{c^2}\right)}\right]^2}(c^2-v^2) = \text{---}$$

$$\cancel{m_0^2c^4} = -2h^2\nu\nu'(1-\cos\phi) + 2h(\nu-\nu')m_0c^2 + \cancel{m_0^2c^4}.$$

$\cancel{2h^2\nu\nu'(1-\cos\phi) = 2}$

$$\cancel{2}h(\nu-\nu')m_0c^2 = \cancel{2}h^2\nu\nu'(1-\cos\phi)$$

$$\left(\frac{\nu-\nu'}{\nu\nu'}\right) = \frac{h}{m_0c^2}(1-\cos\phi).$$

$$\Rightarrow \boxed{\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\phi)} \quad \text{--- (8).}$$

$\boxed{\text{As } \cos\phi < 1 \quad \nu' < \nu}$

Also: $\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\phi)$

$$\Rightarrow (\lambda'-\lambda) = \frac{h}{m_0c}(1-\cos\phi) = \frac{2h}{m_0c}\sin^2\phi/2.$$

$$\boxed{\Delta\lambda = h/m_0c\,(1-\cos\phi) = \frac{2h}{m_0c}\sin^2\phi/2}$$

Change in wavelength of incident radiation

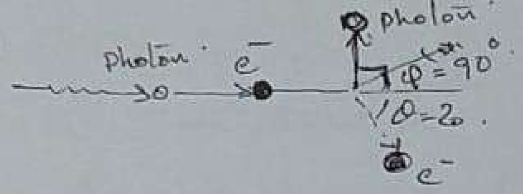
NOTE-
$c = \nu\lambda$.
$\lambda = c/\nu$
Also $\cos\phi = 2\sin^2\phi/2$.

PROBLEM. An X-ray photon is found to have doubled its λ on being scattered by 90°. Find the Energy & λ of incident photon

X-ray photon. Initial wavelength $= \lambda$.
Final " $= \lambda' = 2\lambda$.
Angle of scattering $= \phi = 90^\circ$.

$\lambda' - \lambda = 2\lambda - \lambda = \lambda$.

$\cos\phi = 0$.
$\sin\phi/2 = \sin 45 = 1/\sqrt{2}$



$h = 6.63\times10^{-34}$

4

$m_0 = 9.1\times10^{-31}$ kg

$C = 3\times10^{8}$ m/s

$$\lambda = \Delta\lambda = \frac{2h}{m_0 C}\sin^2\varphi/2 = \frac{\cancel{2}\times 6.63\times10^{-34}}{(9.1\times10^{-31})(3\times10^{8})}\times\frac{1}{\cancel{2}}$$

$$\lambda = \Delta\lambda = 0.02425\times10^{-10}\ m = 0.02425\ \mathring{A}$$

$$\text{Energy } E = h\nu = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3\times10^{8})}{0.02425\times10^{-10}} = \frac{19.89\times10^{-16}}{0.02425}$$

$$E = 8.2\times10^{-15}\ \underline{\underline{J}} = \frac{8.2\times10^{-5}}{1.6\times10^{-19}}\ MeV = 0.051\ MeV$$

million electron volt

10^6

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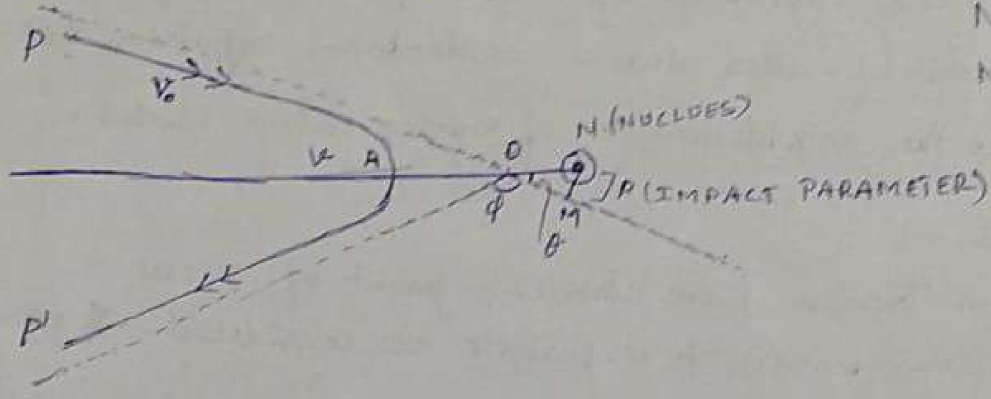
②

Relation between Scattering angle & Impact parameter -



NA = S say.
NM = P

α particle travelling along PAP' (HYPERBOLA), NM = P is impact parameter i.e. the $\perp$ar distance from the nucleus below which α particle gets scattered.

Let the initial vel of α particle $= v_0$.
At pt A. " $= v$
m = mass of α particle.

Law of conservation of momentum -

$$m v_0 P = m v (NA) = m v S \Rightarrow v = \frac{v_0 P}{S} \quad \text{—— ②}$$

At pt. A. KE of α par $= \frac{1}{2} m v^2$
PE " $= \frac{1}{4\pi\varepsilon_0} \frac{(Ze)(2e)}{S}$

At pt. P. KE of α par $= \frac{1}{2} m v_0^2$
PE " $= 0$.

LAW of conservation of energy -

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{S} \quad \text{—— ③}$$

Put value of v from ② in ③,

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m \left(\frac{v_0 P}{S}\right)^2 + \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{S}$$

$$\frac{1}{2} m \left(\frac{v_0 P}{S}\right)^2 = \frac{1}{2} m v_0^2 - \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{S}$$

$\frac{1}{2} m v_0^2 (p/s)^2 = \frac{1}{2} m v_0^2 - \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{s}$ (3)

$p^2 = s^2 \left[1 - \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{\frac{1}{2} m v_0^2 s}\right]$ —— (4).

Substitute $b = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{(\frac{1}{2} m v_0^2)} = b$. —— (5)

we get $p^2 = s^2(1 - b/s) = s(s-b)$ —— (6)

From Co-ordinate geometry of hyperbola eccentricity = $\sec\theta$
where $\theta = \angle NOM$.

f focal distance $NO = a \sec\theta$ where $a = OA = \frac{\text{major axis}}{2}$

$\therefore S = NA = NO + OA = a\sec\theta + a = a(1 + \sec\theta)$ —— (7)

But $NO = a\sec\theta = p\,\mathrm{cosec}\,\theta$. ie $a = \frac{p\,\mathrm{cosec}\,\theta}{\sec\theta} = p\cot\theta$ —— (8)

put value of 'a' in eq (7)

$S = p\cot\theta(1 + \sec\theta) = p\frac{\cos\theta}{\sin\theta}\left(1 + \frac{1}{\cos\theta}\right) = p\frac{\cos\theta}{\sin\theta}\left[\frac{\cos\theta + 1}{\cos\theta}\right]$

$S = \frac{p(\cos\theta + 1)}{2\sin\theta/2 \cos\theta/2} = \frac{p \cdot \cancel{2}\cos^2\theta/2}{\cancel{2}\sin\theta/2\cos\theta/2} = p \cdot \cot\theta/2$ —— (9)

Put value of S in eq^n (6)

$p^2 = p\cot\theta/2\,(p\cot\theta/2 - b)$

$p = p\cot^2\theta/2 - b\cot\theta/2 \Rightarrow b\cot\theta/2 = (p\cot^2\theta/2 - p)$

$\Rightarrow b = \frac{p(\cot^2\theta/2 - 1)}{\cot\theta/2} = p(\cot\theta/2 - \tan\theta/2)$

$b = p\left(\frac{\cos\theta/2}{\sin\theta/2} - \frac{\sin\theta/2}{\cos\theta/2}\right) = p\left(\frac{\cos^2\theta/2 - \sin^2\theta/2}{\sin\theta/2\cos\theta/2}\right)$

$b = 2p\frac{\cos\theta}{\sin\theta} = 2p\cot\theta$ —— (10)

$\tan\theta = \frac{2p}{b}$ —— (11).

Also. from figure. $\varphi = \pi - 2\theta \Rightarrow \varphi/2 = \pi/2 - \theta$

$\cot\varphi/2 = \cot(\pi/2 - \theta) = \tan\theta = 2p/b$. $\boxed{\cot\varphi/2 = \frac{2p}{b}}$ —— (13).

This eq^n — relation bet. impact parameter [and scattering] angle.

(5)

$$N_\varphi = \frac{1}{\cancel{4}} \frac{\cancel{\pi} n N_0 t b^2 \overbrace{\cot \varphi/2}^{\cos\varphi/2 / \sin\varphi/2} \operatorname{cosec}^2 \varphi/2 \, d\varphi}{\cancel{2}\cancel{\pi} r^2 \underbrace{\sin\varphi}_{2 \sin\varphi/2 \cos\varphi/2} d\varphi}$$

$$N_\varphi = \frac{1}{16 r^2} N_0 b^2 n t \operatorname{cosec}^4 \varphi/2 \, d\varphi \quad \text{———} \quad (18)$$

where $b = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{(\frac{1}{2} m v_0^2)}$ ——— (19)

Substituting value of b from (19) in (18)

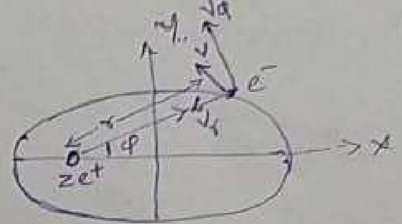
$$N_\varphi = \frac{N_0 n t}{16 r^2} \left[\frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{(\frac{1}{2} m v_0^2)} \right]^2 \operatorname{cosec}^4 \varphi/2 \, d\varphi$$

$$\boxed{N_\varphi = \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{N_0 n t}{16 r^2} \frac{(Ze)^2 (2e)^2}{(\frac{1}{2} m v_0^2)^2} \operatorname{cosec}^4 \varphi/2 \, d\varphi}$$

1915 SOMMERFIELD'S ATOM MODEL (1)

(Sommerfield's Modification of Bohr's Atom Model).

Bohr's simple theory of ⊙lar orbits, inspite of many successes was found inadequate to explain FINE STRUCTURE of Hydrogen atom spectrum.



Assumptions

ELLIPTICAL ORBITS 1) The e^- in the atom moves round nucleus in eliptical orbits instead of ⊙lar orbits.

RELATIVISTIC THEORY 2) The motion of e^- in orbits is relativistic; therefore mass varies with velocity according to relation $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

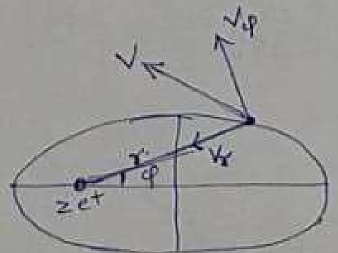
where m = mass of e^-
m_0 = rest mass of e^-
v = vel of e^-
c = vel of light

QUANTIZATION RULE 3) Only those orbits are allowed as stationary states for which.

$$\oint p_i \, dq_i = n_i h \quad (n_i = 1, 2, 3 \text{ —})$$

⟶ [WILSON SOMMERFIELD QUANTISATION RULE]

I ELLIPTICAL ORBITS.



The e^- moving in elliptical orbit its position at any instant can be fixed by polar co-ordinates r & ϕ (radial co-ordinate r, angular co-ordinate ϕ) Tangential velocity v can be resolved, at any instant, into two components —

one along the radius = $V_r = \frac{dr}{dt}$ ie radial momentum

ie radial momentum $P_r = m\frac{dr}{dt}$

& one along line $\perp^r$ to radius called Transverse component

$$V_\phi = r\frac{d\phi}{dt}$$

ie orbital angular momentum $P_\phi = mr^2\frac{d\phi}{dt}$

Wilson - sommerfeld quantisation gives:

$$\oint P_r\,dr = n_r h \quad (1) \qquad \& \qquad \oint P_\phi\,d\phi = n_\phi h \quad (2)$$

$$\& \quad (n = n_r + n_\phi) \quad (3)$$

Evaluation of $\oint P_\phi\,d\phi$

$$\oint P_\phi\,d\phi = n_\phi h \Rightarrow \int_0^{2\pi} P_\phi\,d\phi = n_\phi h$$

$$\Rightarrow P_\phi\,[\phi]_0^{2\pi} = n_\phi h \Rightarrow 2\pi P_\phi = n_\phi h \Rightarrow P_\phi = n_\phi \frac{h}{2\pi} = n_\phi \hbar \quad (4)$$

Evaluation of $\oint P_r\,dr$

$$P_r = m\frac{dr}{dt} \Rightarrow P_r\,dr = m\frac{dr}{dt}\,dr = m\left[\frac{dr}{d\phi}\frac{d\phi}{dt}\right]\left[\frac{dr}{d\phi}\,d\phi\right]$$

$$\Rightarrow P_r\,dr = m\left[\frac{dr}{d\phi}\right]^2\left[\frac{d\phi}{dt}\right]d\phi$$

Also. P_ϕ = angular momentum
= moment of momentum
$= mvr = m(r\omega)r$
$= mr^2\omega = mr^2\frac{d\phi}{dt}$

$$\Rightarrow m\frac{d\phi}{dt} = \frac{P_\phi}{r^2} \quad (5)$$

$$\Rightarrow P_r\,dr = \frac{P_\phi}{r^2}\left[\frac{dr}{d\phi}\right]^2 d\phi \quad (6)$$

Expression for ellipse in polar co-ordinates (7)

$$\frac{1}{r} = \frac{1+\varepsilon\cos\phi}{a(1-\varepsilon^2)} \Rightarrow r = \left[\frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\phi}\right]$$

a = semi major axis.
ε = eccentricity of ellipse.

$$r = \frac{a(1-\varepsilon^2)}{(1+\varepsilon\cos\phi)}$$

differentiating & rearranging

$$\frac{1}{r}\frac{dr}{d\phi} = \frac{r\varepsilon\sin\phi}{a(1-\varepsilon^2)} = \left[\frac{\varepsilon\sin\phi}{1+\varepsilon\cos\phi}\right] \quad (8)$$

Putting eqn (9) in (6) -

$$\oint P_r dr = \oint P_\varphi \left[\frac{1}{r}\frac{dr}{d\varphi}\right]^2 d\varphi = \oint P_\varphi \left[\frac{\varepsilon \sin\varphi}{1+\varepsilon\cos\varphi}\right]^2 d\varphi$$

From eqns (1) & (2)

$$n_r h = n_\varphi \frac{h}{2\pi}\int_0^{2\pi}\left[\frac{\varepsilon \sin\varphi}{1+\varepsilon\cos\varphi}\right]^2 d\varphi$$

$$n_r = \frac{n_\varphi}{2\pi}\left[2\pi\left(\frac{1}{\sqrt{1-\varepsilon^2}}-1\right)\right] \Rightarrow n_r = n_\varphi\left[\frac{1}{\sqrt{1-\varepsilon^2}}-1\right]$$

$$n_r = \frac{n_\varphi}{\sqrt{1-\varepsilon^2}} - n_\varphi \Rightarrow n_r + n_\varphi = \frac{n_\varphi}{\sqrt{1-\varepsilon^2}}$$

Substituting in eqn (3) $(n = n_r + n_\varphi)$

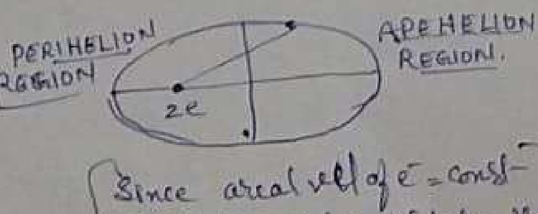
$$n = n_\varphi / \sqrt{1-\varepsilon^2}$$ For ellipse $\sqrt{1-\varepsilon^2} = b/a$

$$n = \frac{n_\varphi}{b/a} \quad \text{ie} \quad \boxed{b/a = \frac{n_\varphi}{n}}$$

This eqn represents the condition of quantisation for the orbits, stating that only those elliptical orbits are permitted for the e^- for which the ratio of major to minor axes is the ratio of two integers.

RELATIVISTIC EFFECT

The velocity of e^- moving in an elliptical orbit varies from maximum to minimum at perihelion to apehelion positions respectively -



[Since areal vel of e^- = const-
Perihelion vel > apehelion vel.]

As the 'v' of e^- nears 'c' a relativistic increase in e^- mass takes place -

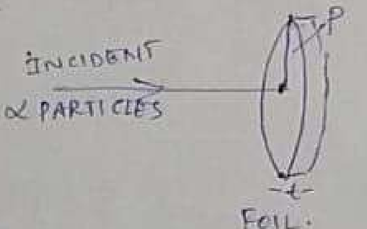
$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

m_0 = rest mass of e^-
v = vel "
c = vel of light.

Smaller p/b, greater is ϕ. (4)

No. of α-particles scattered through ϕ & $\phi + d\phi$

The probability 'q' that an α particle passing within a distance p of any nucleus is



$q = \pi p^2 t n$ —— (14)

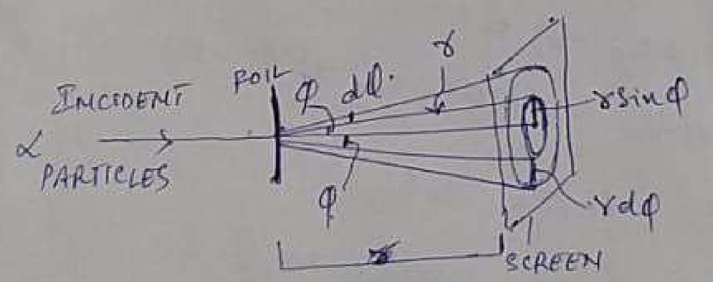
where t = thickness of foil
n = atoms per unit volume.

eqn (13) gives. $p = \frac{b}{2} \cot \frac{\phi}{2}$ —— (15)

ie $\boxed{q = \frac{\pi n t}{4} b^2 \cot^2 \frac{\phi}{2}}$ This gives the probability that an α particle is deflected by an angle equal to or greater than ϕ.

Probability that an α particle is deflected by an angle between ϕ & $\phi + d\phi$ is

$$dq = \frac{1}{4} \pi n t \, b^2 \cot \frac{\phi}{2} \operatorname{cosec}^2 \frac{\phi}{2} \, d\phi \quad \text{—— (16)}$$



If N_0 = Total number of α particles incident normally on the foil. Then, no. of α particles deflected between ϕ & $\phi + d\phi$ will be.

$$dN = N_0 \, dq = \frac{1}{4} \pi n t \, b^2 N_0 \cot \frac{\phi}{2} \operatorname{cosec}^2 \frac{\phi}{2} \, d\phi \quad \text{—— (17)}$$

These dN particles will fall on area

$$2\pi (r \sin\phi)(r \, d\phi) = 2\pi r^2 \sin\phi \, d\phi$$

Hence no. of α particles striking unit area of the Screen in the direction between ϕ & $\phi + d\phi$ will be —

This slows down motion of e^- which slows down the motion & lowers its energy to the total energy.

$$E = \frac{mz^2e^4}{8\epsilon_0^2 h^2}\left[\frac{1}{n^2} + \frac{z^2e^4}{4\epsilon_0^2 c^2 h^2}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)\frac{1}{n^4} + \cdots\right].$$

$$E = \frac{mz^2e^4}{8\epsilon_0^2 h^2 n^2}\left\{1 + \frac{\alpha^2 z^2}{n}\left[\frac{1}{n_\phi} - \frac{3}{4n}\right]\right\}$$

Here $1 + \frac{\alpha^2 z^2}{n}\left[\frac{1}{n_\phi} - \frac{3}{4n}\right]$ is SOMMERFIELD RELATIVITY CORRECTION

where

α = FINE STRUCTURE CONSTANT

$\alpha = \frac{e^2}{2\epsilon_0 hc} = \frac{1}{137}$

$= 7.28 \times 10^{-3}$

This shows that energy of e^- depends on n & n_ϕ also. For a given total quantum number 'n' different energies are obtained for the different orbits corresponding to different values of n_ϕ. Thus the relativistic correction in e^- mass results in splitting up of E levels which increases the number of possibilities of quantum jumps for an e^- in an atom consequently giving rise to fine structure of single spectral lines

SOMMER FIELD'S SELECTION RULE

To explain fine structure of spectral lines sommerfield's selection rule suggests only those transitions can take place between the orbits for which the azimuthal quantum number changes by +1 or -1 ie $\Delta n_\phi = \pm 1$

UNIT-II

WAVE PARTICLE DUALITY

It is a concept in quantum mechanics where every entity in nature may be assigned particle as well as wave nature.

(i) Light - wave nature evidenced by Interference, Diffraction Polarisation.
Particle nature evidenced by - BBR, Photo Elec Effect compton effect --

(ii) Electron - (e^-) - wave nature evidenced by David Germer exp.

(iii) The concept of Plank's Hypothesis & wave particle duality led to the generation of new ideas such as uncertainty principle & quantum statistics

COMPARE WAVES & PARTICLES

WAVE	PARTICLE
1) It is delocalised or spread out in space.	It occupies a well defined position.
2) waves can interfere with each other	Particles do not interfere with each other.
3) It cannot be counted	It can be counted.
4) when two/more waves are present in the same region the resultant wave will be larger/smaller compared to the individual waves.	Two/more particles present in the same region, sum of particles equal to sum of number of individual particles.

PROB Body $m = 0.55\,g$ $v = 3.5\times10^5$ cm/s Find De Broglie λ associated with it

$$\lambda = \frac{h}{mv} = \frac{6.625\times10^{-34}}{0.55\times10^{-3}\times3.5\times10^{5}\times10^{-2}} = \boxed{3.44\times10^{-34}\,m}$$

DE-BROGLIE MATTER WAVES

The ideas that prompted De Broglie to propose concept of matter waves are as below —

1) NATURE IS SYMMETRICAL.

2) RADIATION (LIGHT) is wave like (Interference, Diffraction, Polarisat(i)) & particle like (BBR, PE Effect, COMPTON effect)

3) Hence Matter must also be dual nature. To understand behaviour of matter completely, certain wave aspects should be attributed to material particles.

4) $\bar{e}$ present in stable orbits should be attributed with wave characteristics ie while finding the path of e^- they should be treated as waves.

PROB Calculate De Broglie λ of He ~~atom~~ neutron in a furnace of $T = 400K$

Given Planks const $h = 6.6 \times 10^{-34}$ J.s & Boltzmann const $K = 1.38 \times 10^{-23}$ J/K

$m_{neutron} = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{\sqrt{3mKT}} = \frac{6.6 \times 10^{-34}}{\sqrt{[3 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 400]}} = \boxed{1.26\ \mathring{A}}$$

PROB FIND DE BROGLIE λ of $\bar{e}$ moving $v = 3/5\, c$ given $m_0 = 9.1 \times 10^{-31}$ kg.

$$\lambda = \frac{h}{mv} = \frac{h}{\frac{m_0}{\sqrt{1 - v^2/c^2}} \times v} = \frac{h\left[\left\{1 - \left(\frac{v^2}{c^2}\right)\right\}^{1/2}\right]}{m_0 c \times v/c} = \frac{6.6 \times 10^{-34}\left[1 - (3/5)^2\right]^{1/2}}{9.1 \times 10^{-31} \times 3 \times 10^8 \ (3/5)}$$

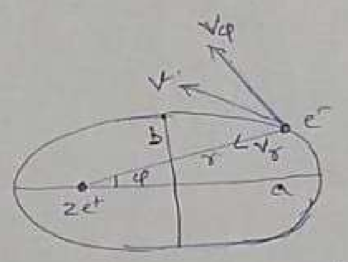
Here $h = 6.6 \times 10^{-34}$ J.s.

$m_0 = 9.1 \times 10^{-31}$ kg,

$C = 3 \times 10^8$ m/s.

$\lambda = 0.032\ \mathring{A}$ $\quad [1\ \mathring{A} = 10^{-10}\ m]$

Derive an expression for total Energy of e^- in an elliptical orbit?



Energy = KE + PE

$\rightarrow$ PE = $-\dfrac{(Ze)(e)}{4\pi\varepsilon_0 r^2}$

$= -\dfrac{Ze^2}{4\pi\varepsilon_0 r^2}$ where Ze = charge on nucleus.
e = e^- charge.
r = Dis bet nucleus & e^-
= Radius of orbit.

KE $\longrightarrow$ Radial vel = $v_r = \dfrac{dr}{dt}$.

Transverse vel = $v_\varphi = r\dfrac{d\varphi}{dt}$

$KE = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}m\left(r\frac{d\varphi}{dt}\right)^2$.

But. P_r = radial momentum = $m\dfrac{dr}{dt}$

& P_φ = Transverse momentum = $mr^2\dfrac{d\varphi}{dt}$ ---- [momentum = m × vel. vel = $v = r\omega$. ang - momentum = m×r×ω. = moment of momentum = $mv \times r$ = $mr\omega \times r$ = $mr^2\omega$ = $mr^2\dfrac{d\varphi}{dt}$

K. Energy = $\dfrac{P^2}{2m} = \dfrac{\left(m\frac{dr}{dt}\right)^2}{2m} + \dfrac{\left(mr^2\frac{d\varphi}{dt}\right)^2}{2mr^2}$

[mr^2 is moment of Inertia. In angular motion mass is replaced by $I = mr^2$.]

$KE = \dfrac{P_r^2}{2m} + \dfrac{P_\varphi^2}{2mr^2}$

$E = KE + PE = \dfrac{-Ze^2}{4\pi\varepsilon_0 r} + \dfrac{P_r^2}{2m} + \dfrac{P_\varphi^2}{2mr^2}$

solving which we get.

$E = \dfrac{-me^4Z^2}{8\varepsilon_0^2h^2}\left[\dfrac{1}{(n_r+n_\varphi)}\right]^2 = \dfrac{-me^4Z^2}{8n^2\varepsilon_0^2h^2}$ (Same as bohr's energy).

Conclusion - Theory of elliptical orbits does not introduce new energy levels.

[note $\rightarrow$ It is the relativistic correction of e^- mass which introduces new E levels & explains fine structure of spectral lines.]

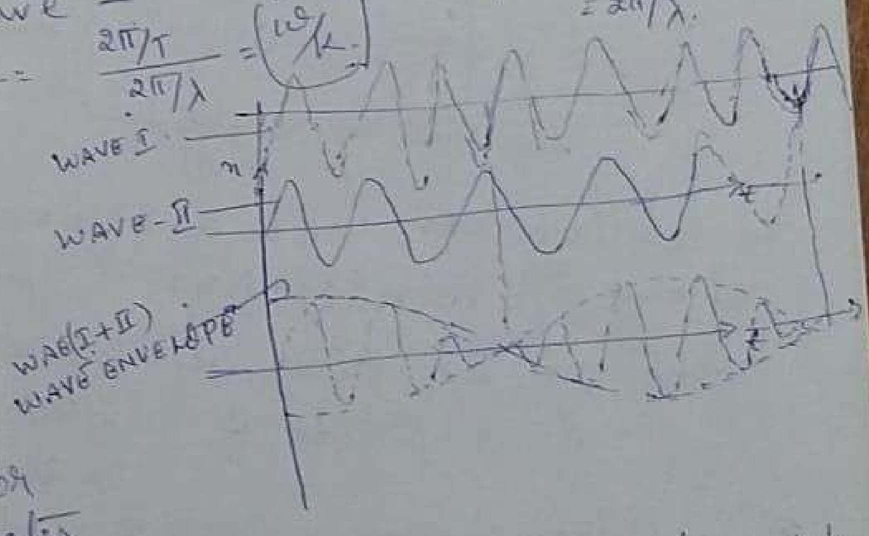
Explain the concept of phase vel & group vel^2.

PHASE VELOCITY

It is the rate at which the phase of the wave propagates in Space - This is the velocity at which the phase of any one frequency component of a group of waves travels. It is equivalent to saying that it is the path diff bet two points on a wave in same phase divided by time taken for it ie Time period of wave -

$$V_p = \frac{dx}{dt} = \frac{\lambda}{T} = \frac{2\pi/T}{2\pi/\lambda} = \boxed{\omega/k}$$

ω = ang. freq = $2\pi/\lambda$.
K = wave number = $2\pi/\lambda$.



GROUP VELOCITY

It is the velocity with which the overal group of waves ie the envelope of wave amplitudes known as the modulation or envelope of waves - propagates through space - It is the velocity with which the energy in the group is transmitted. It is also defined as the rate of variation of angular frequency with wave number K.

$$\boxed{V_g = \frac{d\omega}{dK}}$$

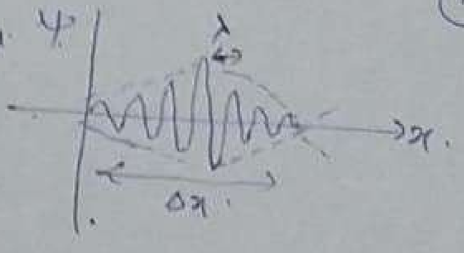
If $\boxed{\omega \propto K, \text{ then } V_g = V_p}$ or else the wave gets distorted as it propagates

This happens for Electromagnetic waves moving in vacuum.

Here $\boxed{\Delta x \ge \frac{1}{\Delta K}}$ ΔK = uncertainty in propagation const $K = 2\pi/\lambda$, wave number

$\Delta x \, \Delta K \ge 1$

$\Delta x (\Delta K \hbar) \ge \hbar$



Now Heisenberg's uncertainty principle

$\Delta x \, \Delta p \ge \hbar/2$

$\therefore \Delta p = \Delta K \hbar = \frac{h}{2\pi} \times \frac{2\pi}{\Delta\lambda} = h/\Delta\lambda$

$\boxed{\Delta p = h/\Delta\lambda}$ is nothing But De Broglie eqn $\lambda = h/p \Rightarrow p = h/\lambda$

$\Rightarrow \Delta p = h/\Delta\lambda$

∴ we can write $\Delta x \, \Delta p \ge \hbar$ or $\Delta x \ge \frac{1}{\Delta K}$ satisfies Heisenberg & DeBroglie eqns.

Also we have $E = h\nu = \hbar 2\pi\nu = \hbar\omega$

$p = h/\lambda = \frac{2\pi\hbar}{\lambda} = K\hbar$

$\hbar = h/2\pi$

$\omega = 2\pi\nu$

$\frac{dE}{dp} = \frac{d}{dp}\left(\frac{1}{2}mv^2\right) = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = p/m = \frac{mv}{m} = v$ (vel of particle).

$\therefore \frac{E}{p} = \frac{\hbar\omega}{K\hbar} = \omega/K \Rightarrow \frac{dE}{dp} = \frac{d\omega}{dK} = v_g$ (vel of ~~particle~~ wave) $= v$ (vel of particle)

Therefore we conclude that group vel v_g = Particle vel. v